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The Fermi Acceleration of Charged Particles

in the Transition Region

Beyond the Magnetosphere

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Abstract

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Recent experiments of IMP1 satellite show the existence of high energy electrons with $E \geq 30$ keV in the transition region between the shock and the magnetosphere. We propose the electrons are locally accelerated by means of a Parker - Wentzel version of the Fermi mechanism. The trapping and acceleration dynamics of a typical electron along the distorted background magnetic field line is studied. It is found that the magnetic bottleneck just outside the shoulders of the magnetopause can reflect the gyrating electron toward upstream of the solar wind, while the large amplitude magnetohydrodynamic wave originated near the shock front can push the electron down stream. The electrons thus trapped gain about 3 - 4 times the energy before they can penetrate the advancing wave. Because of the scattering of small scale magnetic irregularities, a part of the penetrated electrons redistribute their pitch angle and can be trapped again by another advancing wave train. Thus the high energy electrons found in the IMP experiment can be built up by this process in the order of a few minutes.

1. INTRODUCTION

Reports (Ness, et al., 1964) from the measurements of the satellite IMP-1, which was launched in late 1963 by NASA, provides a general picture for the interplanetary magnetic field around the earth. The magnetosphere is abruptly discontinued typically around 10 Re, (earth radius) where the magnetic field B rapidly decreases outward from values of 50 γ to 10 γ and the corresponding variance ΔB increases from the order of $\Delta B/B \sim 0.1$ to $\Delta B/B \sim 1$. This change occurs in a thickness of a few hundred kilometers and may be considered as a discontinuity in gross structures which is called magnetopause. Outside the magnetopause is a transition region where the magnetic field is observed to be turbulent and varies violently. The transition region is separated from the interplanetary field by a collisionless magnetohydrodynamic shock front which is typically located at 13 Re from the center of the earth. Fan, et al. (1964) and Anderson, et al. (1964) have independently detected electrons with energy $E > 30$ Kev in this transition region. Since there can be no prolonged geomagnetic trapping outside the magnetopause, the origin of these high energy electrons is a rather interesting problem. Fan, et al., predicted that they are locally accelerated, whereas Anderson, et al., conceived that they were high energy electrons leaked out from the outer boundary of the magnetosphere.

In this paper we make a theoretical investigation on the feasibility of the mechanism of local acceleration. Because of the characteristic geometry and distorted

structure of the magnetic field near the magnetopause in the transition region, it is plausible that electrons are accelerated to such high energy locally by means of Fermi mechanism. Fermi mechanism was first proposed by Fermi (1949) to explain the origin of cosmic rays, and implies that a charge will gain energy when it is trapped between two magnetic bumps moving toward each other where the particle collides with each bump and reflects back and forth many times. Different versions based on the same basic concept have been considered by various authors. Parker (1958a) considered the case where the moving bumps are hydromagnetic shocks which run across each other. Recently, Wentzel (1963) extended Parker's theory to shock moving on a non-uniform background magnetic field.

The general physical picture of the transfer of energy in the proposed model is as follows: The solar wind carries the interplanetary magnetic field with it, and its magnetohydrodynamic interaction with the geomagnetic field forms the shock front because of the blocking of geomagnetic field against the solar wind. Also, as the dynamic pressure of the solar wind is never exactly constant in time, the small pressure disturbance generates hydromagnetic waves behind the shock front and propagates toward the magnetopause. Because of the low density, the small amplitude waves suffer no dissipation until they catch up with each other and build up to a large amplitude wave pulse. The argument used here is somewhat similar to that of Parker (1958b), who has shown that the flow of solar wind against the geomagnetic field should theoretically lead to hydromagnetic waves propagating down the magnetosphere with $\Delta B/B \sim 1$. The large fluctuations observed by all recent measurements in the transition region seem to indicate that these large amplitude magnetic waves are very likely to occur

as predicted by theory. We shall show in Sections 2 and 3 that because of the geometry of the transition region and magnetopause, under plausible conditions charged particles can be accelerated by these large amplitude waves continuously via the Parker-Wentzel version of Fermi mechanism. This acceleration process is stopped when the cyclotron radius of the particle becomes comparable in size either to the thickness of the large amplitude wave or to the dimension of the bottleneck part of the magnetic field (Fig. 1a). In the former case the Fermi acceleration no longer works because the pulse can no longer trap the particle, while in the latter case the particle cannot reflect back from the bottleneck, it either leaks into magnetosphere or moves to the downstream of the dark side of the earth. Note that the thickness of such hydromagnetic wave pulse is of the order of proton cyclotron radius. Thus, only the electrons not the protons can be trapped and accelerated effectively in the Fermi process. The absolute upper limit of the energy of the accelerated electrons is a few Mev at which the electron's cyclotron radius becomes comparable to the cyclotron radius of a thermal proton (a few Kev). As shown in the following sections, this upper limit can, however, hardly be reached because of the short lifetime of the electrons spent in this transition region.

2. GENERAL DESCRIPTION OF THE ACCELERATION PROCESS

Consider a series of large amplitude waves propagating toward the magnetopause.

We may express the total field strength as

$$B(Z, t) = B_0(Z) + \sum_i H_i(Z - Z_i - u_i t) + \sum_j b_j(Z, t) \delta(Z - Z_j) \quad (1)$$

which contains three parts: $B_0(Z)$, $H_i(Z - Z_i - u_i t)$, and $b_j(Z, T) \delta(Z - Z_j)$.

Here Z is the distance measured downstream from shock front along the line of force. $B_o(Z)$ is the large-scale background magnetic field, whose strength in general increases as Z increases because of the compression of the field, which possesses a magnetic bottleneck or maximum next to magnetopause, either at A or at B as indicated in Fig. 1a. B_o is a slowly varying function of time. In addition to B_o , small scale magnetic irregularities with amplitude $b_j(Z, t)$ are located at various position Z_j 's. Those irregularities are due to current loops produced by the turbulent motion of local plasma. $H_i(Z - Z_i - u_i t)$ is the magnetic pulse propagating outward from the shock front with the peak position Z_i at $t = 0$. And u_i is the parallel (to the line of force) component velocity of the traveling magnetic pulse. The general feature of such a field is shown in Fig. 1b which is roughly what the accelerating electron sees as it gyrates toward one of the magnetic bottlenecks.

We assume that the time variation of all fields considered in this paper is insignificant within the time interval of one Larmor period of electron ($\sim 10^{-3}$ sec). The motion of the particle is then guided adiabatically by the large-scale field and the magnetic pulse, but simultaneously scattered inadiabatically by the small-scale irregularities. As the magnetic pulse moves downstream the local electrons with pitch angle

larger than $\tan^{-1} \left[\frac{B_o(Z)}{H_i(Z)} \right]^{1/2}$ collide with one of the advancing pulses and bounce

back to move toward the magnetic bottleneck. As the particles move into the stronger field, the longitudinal (parallel to the field line) kinetic energy $W_{||}$ of the particle is converted into transverse kinetic energy W_{\perp} . Then most of them reflect back and collide with the pulse again. The rest of them with large initial pitch angles may,

however, overcome the magnetic bottleneck and follow the line of force to leak into the dark side of the earth.

Each collision with the advancing pulse only increases W_{\parallel} but because the pulse is moving toward the stronger field, a part of this longitudinal energy is transformed simultaneously to W_{\perp} . Hence, the pulse feeds both longitudinal and transverse energy into the trapped electron. The feeding process continues itself until the electron gains enough longitudinal energy to penetrate the advancing pulse. Then the electron rolls over the crest of the same magnetic pulse and moves away from the stronger field region. Note that in this process both longitudinal and transverse components of the kinetic energy increase. But the increment in a single trip i.e. the net gain before the particle penetrates the advancing pulse is limited by the structure and magnitude of the magnetic field in the transition region, where in the quiet time of the sun the energy gain is probably less than 5 times its initial energy, and while in the time of geomagnetic storm it is probably less than 10 times (see Sec. 3, Equ (14)). After the electron penetrates the first pulse and moves away from the bottleneck, its transverse kinetic energy is being converted again into longitudinal kinetic energy because of the decrease of field strength. Without the presence of small scale irregularities, the particle will penetrate successive pulse and is unlikely to be trapped but escapes out of shock front into interplanetary space. But the field irregularities b_j scatter the electrons and randomize their energy distribution. The net transfer of the electron's parallel component energy to its transverse component energy, because of the scattering of field irregularities, is proportional to the degree of Anisotropy of the electron's energy distribution between its two components (Parker 1964). When the particle is trapped ahead of the pulse, this scattering effect is insignificant because the pitch angle of the particle varies from less than 45° after the collision with the pulse to near 90° when it is about to be reflected back by the magnetic bottleneck, and the energy transfer between two components

tends to average out between successive collisions. Thus, the net gain of either W_{\parallel} or W_{\perp} due to scattering is small. But as the particle moves away from the bottleneck after penetrating pulses, W_{\parallel} becomes much larger than $1/2 W_{\perp}$ and the pitch angle becomes consistently less than 45° and the conversion of W_{\perp} due to scattering becomes effective.

As a result, the scattering process reduces W_{\parallel} and increases W_{\perp} . So the condition $\frac{W_{\perp}}{W_{\parallel}} > \frac{B}{H}$ may be established before the particle escapes from the transition region to interplanetary space. Then the particle is trapped again by the wave train of advancing pulses and pushed toward the stronger field.

The upper limit of energy gain from the repeated process of acceleration and redistribution is estimated to be 1 Mev. Electrons with such energy have Larmor radius of the same order to that of a Kev proton, which is the scale of the magnetic pulse. However, the electrons can rarely reach such energy since they are likely to be either pushed through the magnetic bottleneck into the dark side of the earth or escape to the interplanetary space before reaching this upper limit of energy.

3. MATHEMATICAL FORMULISM

The process described in Sec. 2 can be separated into two steps:

(1) The acceleration of charged particles when they are trapped ahead of an advancing magnetic pulse and bounce back and forth between the pulse and the bottleneck of $B_0(Z)$.

(2) The redistribution of the energy when the particle moves away from magnetopause.

For the purpose of mathematical convenience, we will consider them separately.

A. Acceleration Process

The motion of a trapped electron which shuttles between the magnetic pulse and magnetic bottleneck is guided adiabatically by B_0 , but scattered nonadiabatically by b_j 's.

The equation of motion in an adiabatic varying magnetic field is well known:

$$\frac{\partial}{\partial t} (m V_{||}) = - \mu \frac{\partial B}{\partial z} \quad (2)$$

Here $V_{||}$ is the parallel component velocity of the particle and $\mu = \frac{W_{\perp}}{B}$ is the

magnetic moment and is an adiabatic invariant. B includes only B_0 and pulse field. The nonadiabatic scattering of a charged particle by magnetic irregularities was treated by

Parker (1964). He considered two examples of irregularities with shape $b = \delta B_0 \left(\exp - \frac{z^2}{d^2} \right)$

and $b = \delta B_0 \frac{z}{d} \left(\exp - \frac{z^2}{d^2} \right)$, respectively. Here δ is the relative amplitude and d the characteristic length of the irregularity. The transfer of energy from one component to another after passage through the irregularity is proportional, respectively, to

$$\delta \frac{d}{\rho} \left(\exp - \frac{d^2}{4\rho^2} \right) \text{ in the first case and to } \delta \left(\frac{d}{\rho} \right)^2 \left(\exp - \frac{d^2}{4\rho^2} \right) \text{ in the second case,}$$

where ρ is the cyclotron radius of the particle. It is evident that the scattering of the particle has a maximum when d is of the order of ρ and diminishes as d becomes either

large or small in comparison to ρ . Thus, we may write the net transfer of a particle's parallel component energy to its transverse component energy (within a distance Δz) due to the scattering of

irregularities as:

$$\frac{(\Delta W_{\perp})_T}{W} - \frac{-(\Delta W_{||})_T}{W} = \eta \left(\frac{2 W_{||} - W_{\perp}}{W} \right) \Delta z \quad (3)$$

where $W = W_{||} + W_{\perp}$ is the total energy, η is the scattering coefficient. If we

take Parker's first example as typical shape of the irregularities, η is given by

$$\eta = \sum_j \frac{b_j}{B_0} \left(\frac{d_j}{\rho} \right) \left(\exp - \frac{d_j^2}{4\rho^2} \right)$$

Here b_j and d_j are the amplitude and characteristic length of the j^{th} irregularity. The summation is to cover all the irregularities within one unit distance along the path of the particles. The values of η depends on the number density and the amplitude of the small scale irregularities but does not depend on the energy distribution of the particles.

Combination of Equations (2) and (3) give the equation governing the change of energy of an electron shuttles between the magnetic pulse and magnetic bottleneck.

$$\frac{dW_{||}}{dZ} = - \frac{W_{\perp}}{B_0} \frac{dB_0}{dZ} + \eta (W_{\perp} - 2W_{||}) \quad (4)$$

$$\frac{dW_{\perp}}{dZ} = \frac{W_{\perp}}{B_0} \frac{dB_0}{dZ} + \eta (2W_{||} - W_{\perp}) \quad (5)$$

Solutions of Equations (4) and (5) can be shown to be

$$W_{\perp}(Z) = \frac{B_0(Z)}{B_0(Z_0)} W_{\perp}(Z_0) f(\eta, Z_0, Z) \quad (6)$$

$$W_{||}(Z) = W_{||}(Z_0) + W_{\perp}(Z_0) \left[1 - \frac{B_0(Z)}{B_0(Z_0)} f(\eta, Z_0, Z) \right]^{(7)}$$

where the scattering function

$$f(\eta, Z_0, Z) = \left(\exp - \int_{Z_0}^Z 3\eta dx \right) + 2B_0(Z_0) \csc^2 \theta_0 \left(\exp - 3 \int_{Z_0}^Z \eta dZ \right) \int_{Z_0}^Z \frac{\eta \left(\exp \int_{Z_0}^x \eta dx' \right)}{B_0(x)} dx \quad (8)$$

measures the net transfer between two components of energy due to scattering. The path of integration in Equation (8) follows the particle's trajectory from Z_0 to Z . Equations (6) and (7) may be compared to the well-known fact that the longitudinal motion of a charged particle in adiabatic varying field is equivalent to that of motion in a one-dimensional potential $\phi = \mu B$. In the present case, μ is not a constant but equal to $\mu(Z_0)f(\eta, Z_0, Z)$.

Detailed calculation including the scattering function $f(\eta)$ is carried out in Appendix I, which gives the energy gain and pitch angle change of an electron during a trip riding ahead of the magnetic pulse. These quantities can be explicitly evaluated only after the form of $B_0(Z)$ is given. As an illustration, the case that the strength of $B_0(Z)$ increases exponentially with Z is shown in the Appendix. However, the actual field in the transition region is much more complicated and obviously does not allow such sophisticated treatment. To be realistic, we assume that the scattering effect is unimportant and can be neglected in the calculation of energy gain during the acceleration process.

The physical reason for this point has already been mentioned in Sec. 2; i.e., the pitch angle varies from less than 45° to more than 45° after each reflection from pulse as the particles move downstream and the energy transfer between two components due to scattering tends to cancel each other between successive collisions. Calculations carried out in Appendix I for small η support this argument mathematically.

The neglect of scattering effect greatly simplifies the problem. The successive collisions with the pulse does not necessarily increase longitudinal kinetic energy of the

particle. As shown in Appendix II, such an increase in longitudinal kinetic energy occurs

only when $\frac{d^2 B_o(Z)}{dZ^2} > 0$. In fact, for a convexly varying magnetic field, all the

energy gain of the particle through collisions with the pulse is transferred to its transverse component. The pitch angle increases monotonically as the particle moves into the stronger field. The particle cannot penetrate the pulse and eventually, after several collisions, it is pushed over the bottleneck and leaks out of the field. However, in a field under consideration, the particle moves from a turbulent field region toward a region next to the magnetosphere where the field strength increases monotonically. Thus, $\frac{dB_o(Z)}{dZ}$ increases with distance. Both the longitudinal and the transverse components of energy increase. The change of pitch angle depends on the ratio of the rate of the field strength change to the rate of energy gain along the path of the particle. If the rate of energy gain due to collisions is larger than the rate of field strength increase, most of the energy gain will be stored in the longitudinal component, and the particle is likely to penetrate through the pulse peak at Z_p and moves to the outer part of the transition region. The condition that a particle penetrates a pulse is

$$W_{\parallel}(Z_p) \cong \frac{W_{\perp}(Z_p)}{B_o(Z_p)} H(Z_p) \quad (9)$$

However, the collision with the pulse does not change μ . Thus, without scat-

tering $\mu = \frac{W_{\perp}(Z_p)}{B_o(Z_p)}$ is a constant. Let Z_1 be the point where the particle was first

trapped by the pulse. We have from the adiabatic condition,

$$W_{\perp}(Z_p) = \frac{B_o(Z_p)}{B_o(Z_1)} W_{\perp}(Z_1) \quad (10)$$

$$W_{\parallel}(Z_p) \approx \frac{H(Z_p)}{B_o(Z_1)} W_{\perp}(Z_1) \quad (11)$$

On the other hand, the particle can be trapped by the pulse at Z_1 only if

$$\frac{W_{\perp}(Z_1)}{W_{\parallel}(Z_1)} \cong \frac{B_o(Z_1)}{H(Z_1)} \quad (12)$$

Combination of Equations (10), (11) and (12) gives

$$W(Z_2) \approx \frac{B_o(Z_p) + H(Z_p)}{B_o(Z_1) + H(Z_1)} W(Z_1) \quad (13)$$

Equation (13) gives the energy gain of the particle in a single trip. This quantity is limited by the extent of the variation of the background field B_o and by the strength of the magnetic pulse H . Consider Z_1 to be some point in the outer transition region and Z_p to be some point near the shoulder of the magnetopause (the bottleneck part). In consistency with the measurements of IMP-1, we may assume $B_o(Z_1) \sim 6 \gamma$, $H(Z_1) \sim 6 \gamma$, $B_o(Z_p) \sim 30 \gamma$, and $H(Z_p) \sim 6 \gamma$. This gives approximately

$$W(Z_p) \approx 3W(Z_1) \quad (14)$$

Clearly, it is unlikely that electrons trapped only once can be accelerated to 30 Kev.

B. The Redistribution Process

After the particle penetrates the first pulse, it moves away from the magnetic bottleneck. At beginning its transverse kinetic energy is converted into longitudinal kinetic energy because of the decrease of the magnetic field strength. There is no acceleration along the path since the particle can no longer be reflected back by the successive pulses. But because of the increase of the asymmetry of the energy distribution,

the scattering effect becomes more and more efficient and the redistribution process begins.

From Equations (6) and (7) and the condition $\frac{W_{\parallel}(Z_p)}{W_{\perp}(Z_p)} \cong \frac{H(Z_p)}{B_o(Z_p)}$ as the particle penetrates the first pulse, we have at any position Z

$$W_{\perp}(Z) = \frac{B_o(Z) f(\eta, Z, Z_p)}{H(Z_p) + B_o(Z_p)} W \quad (15)$$

$$W_{\parallel}(Z) = \left[1 - \frac{B_o(Z) f(\eta, Z, Z_p)}{H(Z_p) + B_o(Z_p)} \right] W \quad (16)$$

The particle can be trapped again by a second pulse, only if

$$\frac{W_{\parallel}(Z)}{W_{\perp}(Z)} \cong \frac{H(Z)}{B_o(Z)} \quad (17)$$

Combination of Equations (15), (16) and (17) gives

$$f(\eta, Z, Z_p) = \left\{ 1 + [H(Z_p) + B_o(Z_p)] \int_{Z_p}^Z \frac{\eta_e 2 \int^x \eta dx'}{B_o(x)} dx \right\} e^{-2 \int^Z \eta dx} Z_p \cong \frac{H(Z_p) + B_o(Z_p)}{H(Z) + B_o(Z)} \quad (18)$$

EQUATION (18) SERVES AS A CRITERION FOR THE RETRAP OF ELECTRONS,

4. DISCUSSIONS AND CONCLUSIONS

In a recent article, Jokipii and Davis (1964) suggested that these 30 Kev (or higher) electrons may be accelerated outside the shock front by a first-order Fermi mechanism. Field fluctuations carried by solar wind move toward the shock front, electrons are trapped by the fluctuation and reflecting back and forth between the fluctuation and the shock front. As they pointed out, it takes of the order of 200

reflections to accelerate a 1-Kev electron to 50 Kev. Thus, about 90-95 percent of the electrons approaching the two mirror points must be reflected to produce detectable fluxes. This requires extremely efficient randomization process produced by magnetic irregularities. In the model we discussed, the electrons are Fermi accelerated inside the transition region near the magnetopause. The efficiency of acceleration in this region is of the same order as that in the "outside" region. But the requirement for efficient randomization to trap the electrons in the acceleration region is much relaxed here. Because of the increase of field strength, the pitch angle of the electrons can be kept from decreasing even without efficient randomization of irregularity scattering, and a long duration of trapping is achieved. Efficient randomization occurs only when the electrons are not trapped by but roll over the magnetic pulses, the distance traveled and the irregularities encountered are many times that of a single reflection. Therefore, the chance that electrons will be trapped again is increased. Since an electron gains about three times its initial energy in a single trip, it takes of the order of $\log_3 \frac{3 \times 10^4}{W_0}$ trips to boost a particle's energy from W_0 ev to 30 Kev. Thus, for a 1-Kev electron it has to be trapped three times while for a 10 ev electron it has to be trapped seven times. To produce detectable fluxes only 10 percent of the electrons need to be retrapped. The time required in this acceleration process can also be estimated. The electrons are trapped by the magnetic pulse somewhere in the transition region and penetrate the pulse near the magnetopause. Thus, the distance traveled by the pulse is 10,000 km. The propagation velocity of the pulse along the field line is 200 km/sec (the Alfvén velocity near the magnetopause). Therefore, in order to reach 30 Kev, electrons with energy W_0 need to stay in the transition region for at least $50 \log_3 \frac{3 \times 10^4}{W_0}$ sec, which is of the order of a few minutes.

As a conclusion of this paper, we would like to list the basic assumptions of the proposed model and the major consequences which can be checked experimentally.

Basic Assumptions

- (1) There are large amplitude waves originated from the shock propagating toward the magnetopause to cause the trapping and bouncing of electrons.
- (2) There are small scale magnetic irregularities to cause the randomization of pitch angles of electrons.
- (3) The magnetic field in the transition region is, in general, stronger near the shoulder of the magnetopause through compression than in the outer portion of the region.

These assumptions are, as explained in the earlier part of this paper, theoretically founded. And they are also compatible with the present experimental knowledge.

Major Consequences

- (1) Only electrons can be accelerated. The upper limit of the accelerated electron's energy is roughly 1 Mev.
- (2) The accelerated electrons tend to bunch together because the trapping regions are ahead of the wave pulse. Also, the acceleration process is more likely to happen in the upper part of the transition region where the average interplanetary magnetic field is under compression. Because the electrons move along the interplanetary field lines (Fig. 1a) which make an angle with the sun-earth line, the electrons are more likely to be reflected back by the bottleneck A than by the bottleneck B in the lower part of the transition region. And one would expect to find more "islands" of high energy electron in the upper region than in the lower region.
- (3) The intensity of the high energy electrons is closely correlated with the degree of disturbance of the magnetic field in the transition region (the evidence of the presence of large amplitude wave). After the transition region is hit by a magnetic storm,

there should be a time lag of the order of a few minutes before any increase in the intensity of high energy electrons.

(4) The accelerated electrons may leak out of the shock front to interplanetary space. They may also be pushed through the bottleneck part of the field and leak to the dark side of the earth.

(5) The angular distribution of the accelerated electrons is anisotropic.

Comparison with the existing experimental measurement indicates good qualitative agreement for the major consequences as listed in (1), (2), (3) and (4). The angular distribution of the high energy electrons, as far as we know, has yet to be measured. Thus, no data are available to check the consequence (5). However, this is probably the most interesting one. According to the proposed mechanism, when the electron is about to penetrate the magnetic bottleneck near the magnetopause at ξ (Cf. Fig. 2),

$W_{\parallel} \approx \frac{H(\xi_1)}{B(\xi_1)} W_{\perp}$ and $\tan^2 \theta(\xi_1) \approx \frac{B(\xi_1)}{H(\xi_1)} > 1$. But after it penetrates the pulse and moves away from the stronger field, most of its energy is transformed to longitudinal component. Conservation of energy gives

$$W_{\parallel}(\xi_2) \approx \frac{H(\xi_1) + B(\xi_1) - B(\xi_2)}{H(\xi_1) + B(\xi_1)} W$$

and

$$\tan^2 \theta \approx \frac{B(\xi_2)}{H(\xi_1) + B(\xi_1) - B(\xi_2)} < 1$$

As it moves further away from the magnetopause into the outer transition region, the scattering with small scale irregularities gradually smears out the asymmetry and $\tan^2 \theta$

approaches unity. Thus, W_{\perp} approaches W_{\parallel} and the energy of the accelerated electron tends to reach isotropy. A measurement of the pitch angle along the field line with satellite should well indicate the correctness of our conjecture.

APPENDIX I

We want to calculate the energy gain and pitch angle change of an electron during a trip riding ahead of a magnetic pulse. The energy of an electron at different positions between two successive collisions is given by Equations (6), (7) and (8) of Section 2.

Now let Z be the position where the electron makes its first collision with the magnetic pulse. $W_{\parallel}(Z_1)$, $W_{\perp}(Z_1)$, V_{\parallel} and θ be the longitudinal energy, the transverse energy, the longitudinal velocity, and the pitch angle of the particle at Z before collision, respectively. ζ_1 be the mirror point after first collision, Z_2 be the position of the particle at the next collision and T be the time duration between the two collisions.

Then from Equations (6) and (7) and the fact that the collision increases the longitudinal energy of the particle by a factor of $\gamma = \left(1 + \frac{2u}{V_{\parallel}}\right)^2$, we have

$$B_o(\zeta_1) = B_o(Z) (1 + \gamma \cot^2 \theta_1) / f(\eta, Z_1, \zeta_1) \quad (1-1)$$

$$T = \oint_{Z_1}^{Z_1+uT} \gamma V_{\parallel}^2(Z_1) \left[1 - \frac{B_o(Z)}{B_o(Z_1)} \sin^2 \theta_1 f(\eta, Z_1, Z) \right]^{-1/2} dZ \quad (1-2)$$

where the integral sign $\oint_{Z_1}^{Z_1+uT}$ means the path of integration is to be taken along the electron's trajectory from Z_1 to ζ_1 , then from ζ_1

$$Z_2 = Z_1 + uT \quad (1-3)$$

$$W_{\perp}(Z_2) = \frac{B_o(Z_2)}{B_o(Z_1)} W_{\perp}(Z_1) f(\eta, Z_1, Z_2) \quad (1-4)$$

$$W_{\parallel}(Z_2) = \gamma W_{\parallel}(Z_1) + W_{\perp}(Z_1) \left[1 - \frac{B_o(Z_2)}{B_o(Z_1)} f(\eta, Z_1, Z_2) \right] \quad (1-5)$$

Equations (1-1) to (1-5) determine implicitly the energy, the pitch angle and the position of the electron at next collision in terms of the same quantities at first collision. The explicit expression, however, may be evaluated only after the form of $B_o(Z)$ is given.

The particle is pushed into stronger field by successive collisions. The energy of the particle at Position Z_n of a later collision is given by

$$W_{\perp}(Z_n) = \frac{B_o(Z_n)}{B_o(Z_1)} W_{\perp}(Z_1) f(\eta, Z_1, Z_n) \quad (1-6)$$

$$W_{\parallel}(Z_n) = W_{\parallel}(Z_1) + W_{\perp}(Z_1) \left[1 - \frac{B_o(Z_n)}{B_o(Z_1)} f(\eta, Z_1, Z_n) \right] + \int_{Z_1}^{Z_n} \frac{2m(u + V_{\parallel}(x))}{\tau(x)} dx \quad (1-7)$$

and

$$W(Z_n) = W(Z_1) + 4 \int_{Z_1}^{Z_n} \frac{W(x) \cos^2 \theta}{\tau(x) V_{\parallel}(x)} \left(1 + \frac{u}{V_{\parallel}} \right) dx \quad (1-8)$$

After a number of reflections, the electron eventually penetrates the pulse at Z_p . The total energy gain during the whole trip is given by

$$\Delta W = \int_{Z_1}^{Z_n} \frac{4W(x) \cos^2 \theta}{\tau(x) V_{\parallel}(x)} \left(1 + \frac{u}{V_{\parallel}} \right) dx \quad (1-9)$$

where the change in pitch angle is given by

$$\frac{\sin^2 \theta_n}{\sin^2 \theta_1} = \frac{1}{1 + \int_{Z_1}^{Z_n} \frac{W_{\parallel}(x) \left(1 + \frac{u}{V_{\parallel}}\right)}{W_{\parallel}(Z_1) T(x) V_{\parallel}(x)} dx} \frac{B(Z_n)}{B(Z_1)} f(\eta, Z_1, Z_n) \quad (1-10)$$

To illustrate the calculations, consider the case that η is a small constant; i.e., the magnetic irregularities are evenly distributed. And $B_o(Z) = B_o(0)e^{Z/L}$, where L is the scale length of the field variation.. Comparison with the actual transition region magnetic field, whose strength increases from a few γ to 30γ near the magnetopause, L should be of the order of 10^4 km. Then the scattering function can be evaluated

$$f(\eta, Z_o, Z) = e^{-2\eta(Z-Z_o)} \left\{ 1 + \frac{\eta}{\eta - \frac{1}{L}} \left[e^{(2\eta - \frac{1}{L})(Z-Z_o)} - 1 \right] \csc^2 \theta_o \right\} \quad (1-11)$$

In the calculation of Equations (1-1) to (1-5), the distance traveled by electrons between the collision point Z_n and mirror point Z_m is short. (Effective Fermi acceleration occurs as the electrons are pushed near the magnetopause, when the number of reflections per unit time increases.) We may write

$$e^{\frac{Z-Z_o}{L}} \approx 1 + \frac{Z-Z_o}{L} \quad (1-12)$$

This is equivalent to assume that the field can be considered linearly varying with Z between two collisions. Also, because η is small

$$e^{\eta(Z-Z_o)} \approx 1 + \eta(Z-Z_o) \quad (1-13)$$

then

$$f(\eta, Z, Z_o) \approx 1 + \eta(Z-Z_o)(\cot^2 \theta_1 - 1) + O(\eta^2, \frac{1}{L^2}) \quad (1-14)$$

Substitute Equation (1-14) into Equations (1-1), (1-2) and (1-3) gives to the lowest order of η and $\frac{1}{L}$

$$\tau_1 = \frac{4L \cot^2 \theta_1}{[1 + L \eta (\cot^2 \theta_1 - 1)]} v_{\parallel}(z_1) \quad (1-15)$$

$$z_2 = z_1 + \frac{4L \cot^2 \theta_1}{1 + L \eta (\cot^2 \theta_1 - 1)} \frac{u}{v_{\parallel}(z_1)} \quad (1-16)$$

$$w_{\perp}(z_2) = \chi w_{\perp}(z_1) \quad (1-17)$$

$$w_{\parallel}(z_2) = \chi w_{\parallel}(z_1) - (\chi - 1)w_{\perp}(z_1) \quad (1-18)$$

where

$$\chi = \left[1 + \frac{2L\eta \cot^2 \theta_1 (\cot^2 \theta_1 - 1)}{1 + L\eta (\cot^2 \theta_1 - 1)} \right] \left[1 + \frac{4 \cot^2 \theta_1}{1 + L\eta (\cot^2 \theta_1 - 1)} \frac{u}{v_{\parallel}(z_1)} \right]$$

Note that when $\eta = 0$, $\chi = 1 + 4 \cot^2 \theta \frac{u}{v_{\parallel}(z_1)}$, And

Equations (1-17) and (1-18) become

$$w_{\perp}(z_2) = \left[1 + 4 \cot^2 \theta_1 \frac{u}{v_{\parallel}(z_1)} \right] w_{\perp}(z_1) \quad (1-19)$$

$$w_{\parallel}(z_2) = \left[1 + \frac{4 u^2}{v_{\parallel}^2(z_1)} \right] w_{\parallel}(z_1) \quad (1-20)$$

Thus, the increase of w_{\perp} is first order in u/v_{\parallel} while the increase of w_{\parallel} is second order

in u/V_{\parallel} . The energy of the electron after a number of collisions can be evaluated by neglecting higher order terms of u/V_{\parallel} in Equation (1-8)

$$W'(Z) = W(Z_1) \left(\exp \frac{Z-Z_1}{L} [\sin^2 \theta + L\gamma(\cos^2 \theta - \sin^2 \theta)] \right) \quad (1-21)$$

which gives the energy gain of the trip ahead.

APPENDIX II

The equation governs the change of longitudinal component energy of a particle in an adiabatically varying field is

$$\frac{dW_{\parallel}(Z)}{dZ} = -\mu \frac{\partial B_o(Z)}{\partial Z} \quad (2-1)$$

Transform the above equation to the frame of reference moving with the advancing pulse

$$\frac{dW_{\parallel}(Z')}{dZ'} = -\mu \frac{\partial B_o(Z' + ut)}{\partial Z'} \quad (2-2)$$

The longitudinal energy change between two successive collisions in the prime system is

$$\Delta W_{\parallel} = -\mu \int_{t=t(Z')} \frac{\partial B_o(Z' + ut)}{\partial Z'} dZ' \quad (2-3)$$

The pulse is stationary in this system, so two collisions occur at the same position. But B_o depends on time and $\frac{\partial B_o(Z' + ut)}{\partial Z'}$ is to be evaluated along the particle's path. Hence although the integral is a closed one it in general does not vanish.

Equation (2-3) can be written as

$$\Delta W_{\parallel} = -\mu \int_{Z_o'}^{\zeta'} [B_o'(Z' + ut_1(Z')) - B_o'(Z' + ut_2(Z'))] dZ' \quad (2-4)$$

where Z_o' and ζ' are the collision position and the mirror position, respectively

$$t_1(Z') = \int_{Z'_0}^{Z' + ut_1} \frac{dx}{\left(\frac{2}{m}\right)^{1/2} \left[W_{\parallel}(Z'_0) - \mu B(x) + \mu B(Z'_0) \right]^{1/2}} \quad (2-5)$$

is the time interval for the particle to travel from Z'_0 to Z' .

$$t_2(Z') = t_1(Z') + 2 \int_{Z' + ut_1}^{\zeta} \frac{dx}{\left(\frac{2}{m}\right)^{1/2} \left[W_{\parallel}(Z'_0) - \mu B(x) + \mu B(Z'_0) \right]^{1/2}} \quad (2-6)$$

is the time interval for the particle to travel from Z'_0 to ζ and then return to Z' .

By applying mean value theorem, Equation (2-4) can be written as:

$$\Delta W_{\parallel} = \mu \int_{Z'_0}^{\zeta'} \left[t_2(Z') - t_1(Z') \right] B''_0(\xi) dZ' \quad (2-7)$$

ξ is some value between $Z' + ut_1$, and $Z' + ut_2$. Since $\zeta' > Z'_0$ and $t_2(Z') > t_1(Z')$, $\Delta W_{\parallel} > 0$ for $B''_0 > 0$ where $\Delta W_{\parallel} < 0$ for $B''_0 < 0$. In the former case the particle gains energy in both longitudinal and transverse components. In the latter case all the energy gain of the particle through collisions with the pulse is transferred to its transverse component, and the longitudinal energy is, in fact, reduced.

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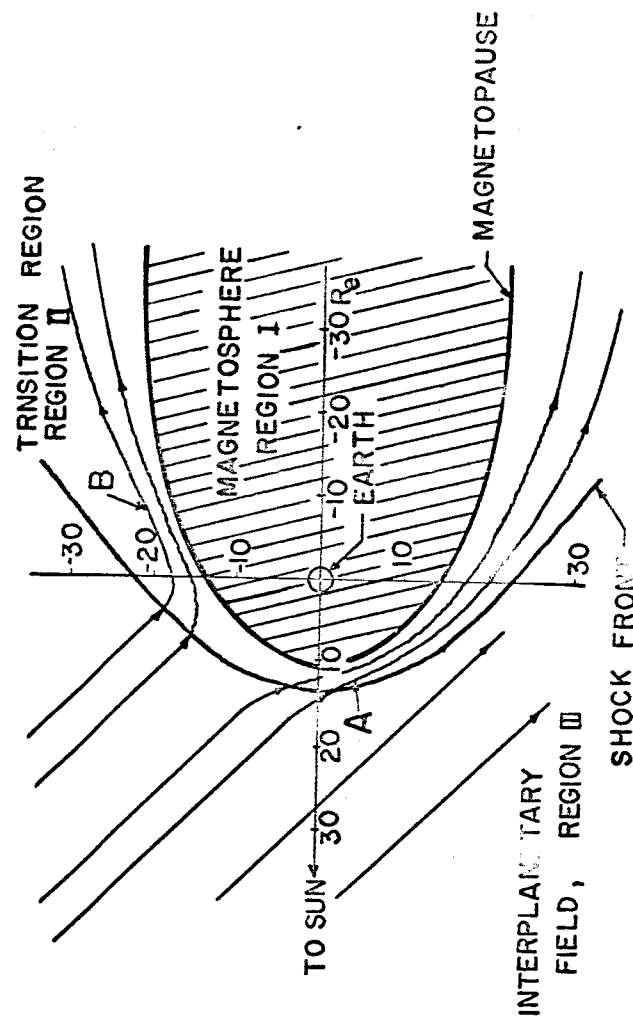


Figure 1a. The structure of magnetic field outside the magnetosphere on the ecliptic plane. Charged particles move along the field line where the line drifts towards the east because of sun's rotation. A and B are bottlenecks of magnetic field.

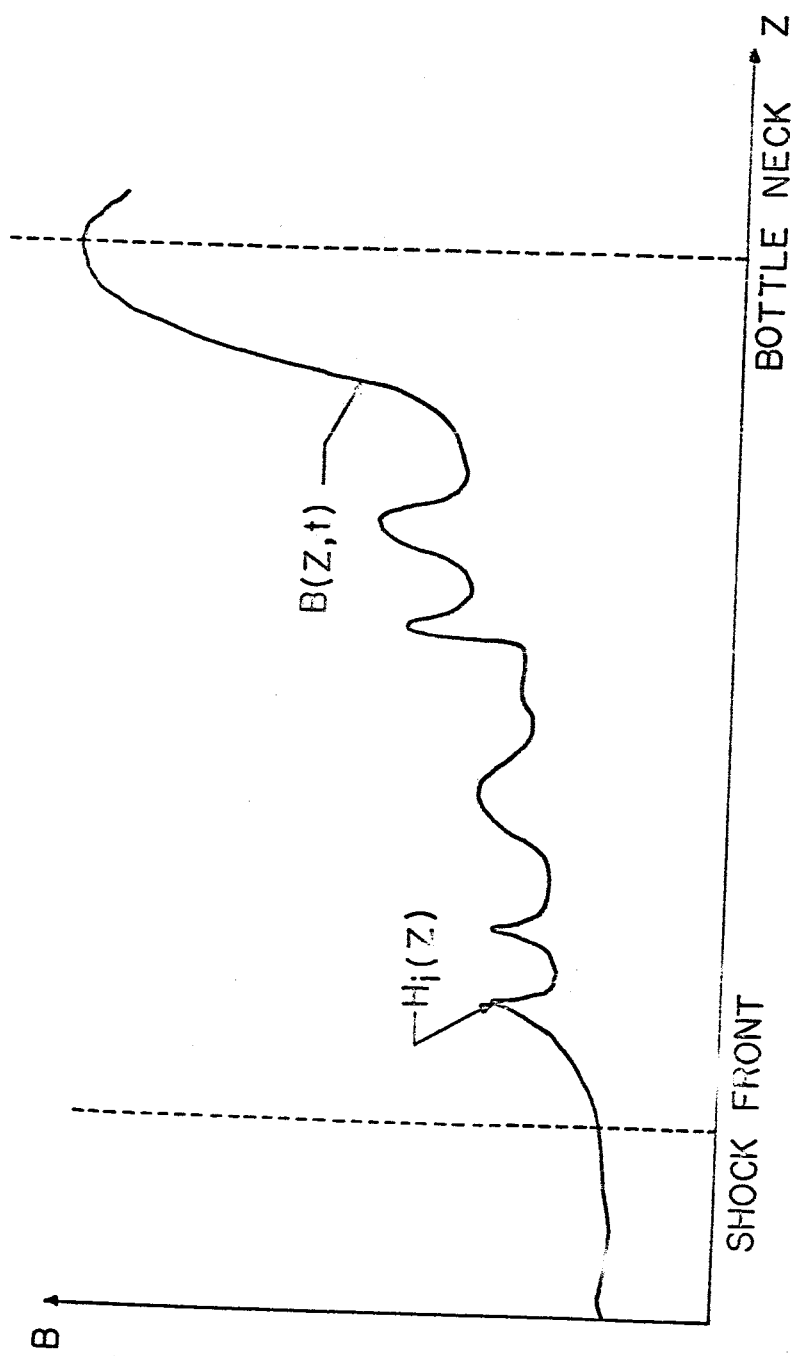


Figure 1b. Total Magnetic field strength $B(Z, t)$ along the magnetic line.

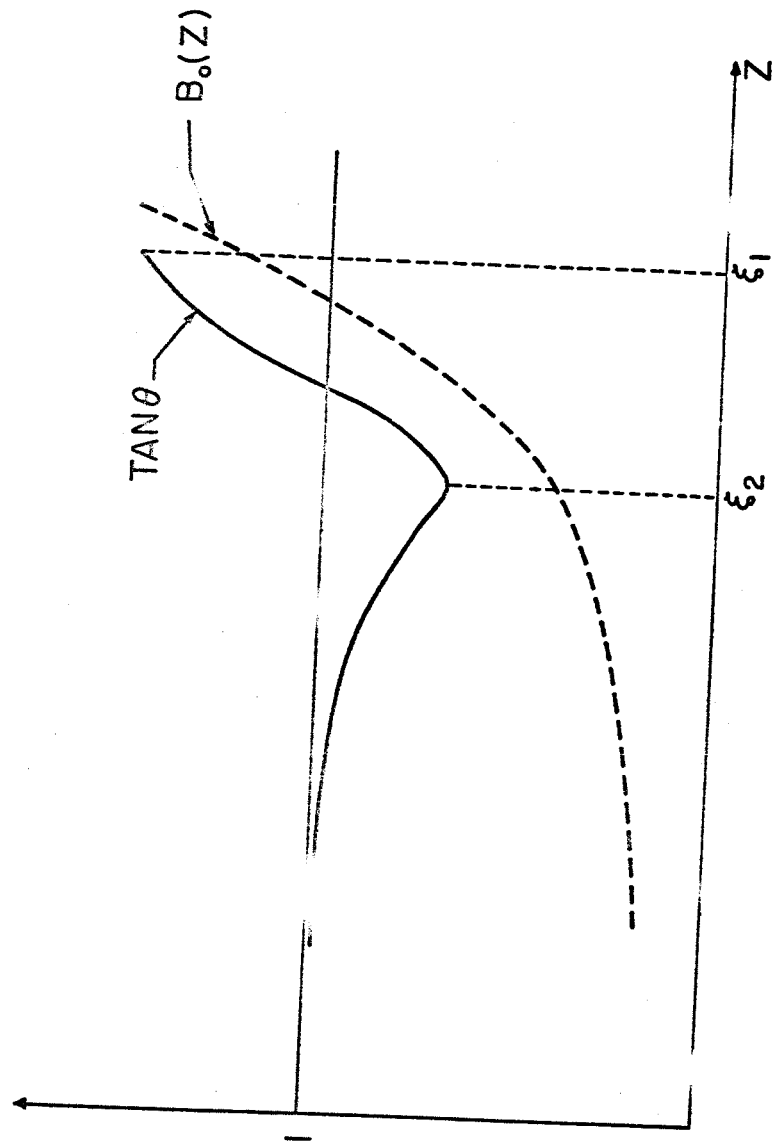


Figure 2. Estimated distribution of pitch angle θ and the background $B_0(Z)$ along the magnetic line.